Space-Time Diffusion Visualization using Bayesian Inference

Research-in-Progress

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Abstract

Retail marketing geography has traditionally employed static gravity models for location analytics based on probabilistic consumer demand. Such models however, provide little insight into the dynamic space-time diffusionary processes and events within a store level trade area (SLTA) that lead up to the end state of market structure equilibrium condition which these models predict. In addition, attempts to display dynamic space-time events in GIS have produced visualizations that are perceptually complex and difficult to easily understand. Further those attempts fail to take advantage of transdisciplinary literature from psychology and brain science that suggest image simplicity and visual familiarity are the key elements to faster perception and better visuospatial understanding of objects, images and visualizations. We address these issues with our visualization model called Avatar. Avatar uses Bayesian inferences to link the Bass model to a spatial allocation methodology which is designed to provide faster and better visuospatial understanding of complex "geo-big-data". We then introduce the steps necessary to create our Avatar, 3-D semaphoric space-time visualization diffusion object.

Avatar was originally designed to visualize an ensemble model consisting of the integration of Bass, Huff and Berry concepts with a Bayesian inference framework for visualization of determinant attributes and demographics. In this way we can map the timed hierarchical diffusions of new innovative products throughout the SLTA as well as any evolving store network. Our approach is empirically supported by 5 years of panel data from the Southern California market. In addition, we provide priors linked to statistical demographic units (i.e. Census Blocks) for spatially allocating and distributing the Bass model forecasts through the SLTA. We conclude that our validated space-time diffusion visualization will potentially strengthen local "location analytics" and extend such analysis methodology across many spatial temporal domains.

Keywords: Location Analytics, Location Intelligence, GIS, 3D, Visualization, Bayes Theorem, Bayesian Inference, Huff Model, Bass Model, Berry Model, Avatar, Space-Time, Trade Area Analysis, Trade Area Structure and Shape, Hierarchical Innovation Diffusion, Polar Azimuthal Equidistant Projection, Discrete Choice Models

Introduction

"As the mass of data generated to understand business problems increases both in terms of variety as well as complexity, better methods are needed to display, communicate and analyze such information" (Huff, 1981).

When Huff published this statement in the fall of 1981, the first fledging IBM-PC had just been introduced and yet Huff's statement today is as timely as it was over 34 years ago. Today increasing quantities of digital information, in various forms and complexity, are accumulating at increasing orders of magnitude, over shorter and shorter periods of time. If this information can be presented in familiar and recognizable visual formats, the literature of psychology and brain science tell us we can leverage the brains powerful cognitive recognition abilities through visualization design, of the "geo-big-data" order of magnitude, resulting in faster perceptual transfers of both meaning and understanding.

In retail marketing geography, going past 2D visualizations, may hold potential for greater understanding of the complex "geo-big-data" of consumption behavior at the micro-analytic (Mason, 1975) urban store level trade area. Understanding the "big picture" of diffusion of adoption (Rogers, 1976) for a trade area as well as the adoption effects of space and time on product life cycles (Bass, 1969), may potentially strengthen a retailer's ability to focus both on store level trade area shapes and structures (Mason, 1975) and also the store level sales processes that drive the adoption of new product innovations across entire, evolving store networks (Berry, 1971).

To this end our study presents a novel approach for visualizing and exploring the space and time dimensions of panel geo-big-data. Such visualizations may offer new insights into the use of Euclidean distance to identify interesting correlations (Bunge, 1962) from a mathematical geography perspective (Goodchild, 2008). Polar azimuthal equidistant projections in geography are not new and date back, over 1000 years, to the Muslim scholar al Biruni (King, 1996) but there are current and familiar examples such as the United Nations (**Figure 1**).

Figure 1. Example of a familiar polar azimuthal equidistant projection

But few, if any examples in the literature, replace the meridians or longitudinal frames of reference with temporal frames of reference. We refer to such an approach as the Avatar concept (see **Figure 2**).

The Avatar was developed jointly as a novel and innovative visualization for a robust ensemble model we also developed to link Bass, Huff and Berry hierarchical innovation diffusion concepts. The ensemble model "introduces a Bayesian framework for inference, visualization and hypothesis testing of…" (Lemey, 2009) innovative product life cycles (e.g. Bass Model) throughout and across store level trade areas (i.e. modified Huff/Bayes), as well as across and throughout evolving networks (i.e. Berry Model).

We begin with a brief review of the history of store trade area analysis and visualization, to provide context to what follows. We then outline the steps needed to build an ensemble space-time trade area visualization model, proceed sequentially through these steps and end with the product, a 3-D printed Avatar model.

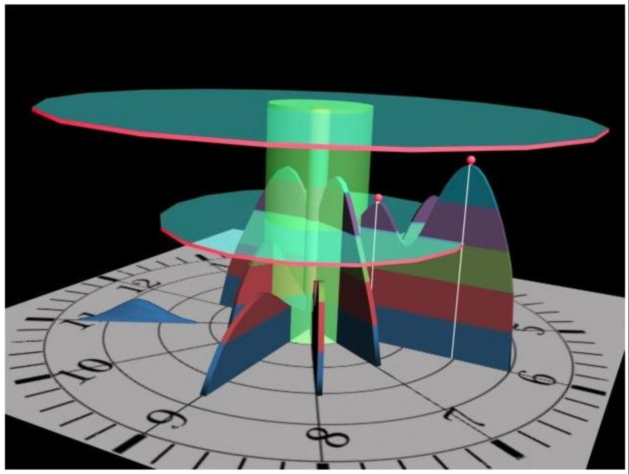


Figure 2. The Avatar Concept – a time-modified polar azimuthal equidistant projection

Discussion

Visualizations of gravity models have been popular in geographic information system (GIS) applications and still dominate retail trade area analysis, location analytics and location intelligence activities. Gravity models are a class of spatial interaction models (Haynes, 1984) that calculate non-subjective, empirically based, probability estimates of static retail trade area demand limits (Huff, 1963).

Their long history and development is depicted in the flow chart found in **Figure 3**.

"When a new phenomenon (bus stop, emergency center, retail store or even advertising campaign) is introduced into an existing spatial market, DISCRETE CHOICE models estimate the eventual market structure equilibrium rather than the PROCESS of attaining it." (Allaway, Black, Richard, Mason, 1994)

A typical example of a trade area analysis visualization can be seen in **Figure 4**, but it provides little if any information about the spatial and temporal differences of neither standard product sales nor innovative new product sales, or any information about the intervening spatial temporal states of hierarchical innovation diffusion evolving throughout the trade area. In other words there is no dynamic, high-resolution, temporal component in Figure 4, much less any visualization of a space-time scenario.

Space-time visualizations in GIS have been attempted (Kwan, 2004) with varying degrees of comprehensibility from the perspective of an educated observer. **Figure 5** shows the effect of joining a functional map with the dimension of time. But it is difficult to sense any temporal context of beginning or end, and once the added explanation of an increase in elevation representing an increase in time is added, the sensory visual perception overloads and the comprehensibility factor for the model reduces significantly for the non-expert, educated observer. In gestalt theory this is because the literal objects presented in the visualization do not in and of themselves relate to any familiar representations of time – for example like a clock face might. The mind's perceptual system, once set for the expectation of geographic objects like maps, cannot easily incorporate other dimensions. Connecting spatial and temporal dimensions in this way fails to take advantage of the extensive body of brain science and psychological literature that clearly indicates that the key to improving object recognition and understanding, associated with human perceptual processes, is simplicity and familiarity.

Other attempts at space-time visualizations have included "clock images" being placed on maps with interpretive "roses" added to facilitate interpretation of the space-time dimensions (see **Figure 6**). It continues to remain remains difficult to envisage.

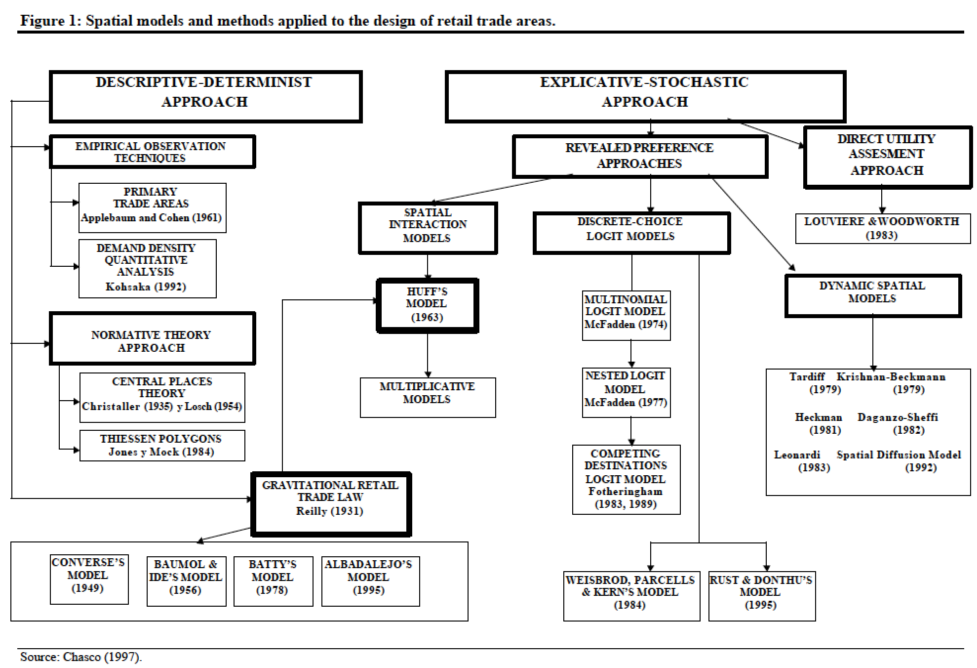
Is there a perceptual-neutral space-time visualization that is easy for anyone to comprehend, regardless of analytic skill and still be able to amplify meaning, while reducing the volumes of very large data sets into simple objects that can easily be understood?

Therefore in this study our key strategy is to reduce wherever possible what we consider noise or confusing unnecessary details; thus we look at only the necessary and sufficient dimensions, space, time, and attributes.

Visualization Research Question

How can dynamic consumer and/or customer areal unit distribution patterns best be visualized in space and time; as "consumers" convert to new "CUSTOMERS" and customers repeat purchasing behavior through the process of adoption during the product life cycles of new innovative products diffusing across Store Level Trade Areas (SLTA) and evolving hierarchical networks of store?

This is the essence of the visualization problem we are addressing.



SOURCE: Spanish trade AREA_viena98.pdf

Spatial Interaction Models applied to the Design of Retail Trade Areas
Coro Chasco Yrigoyen, Jose Vicens Otero. L.R. Klein Institute, Autonoma University of Madrid, Spain
1998 - coro.chasco@uam.es; jose.vicens@uam.es

Figure 3. Spatial Model Genealogy

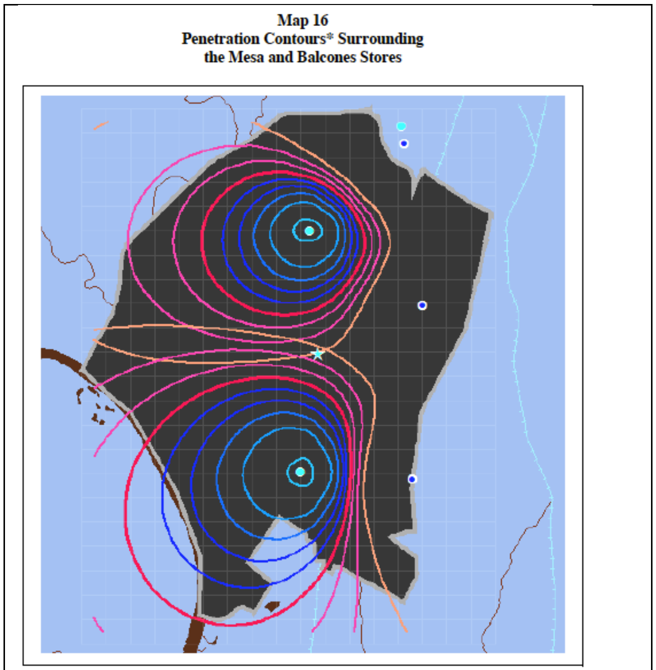


Figure 4. Trade Area Analysis (Huff, 2008)

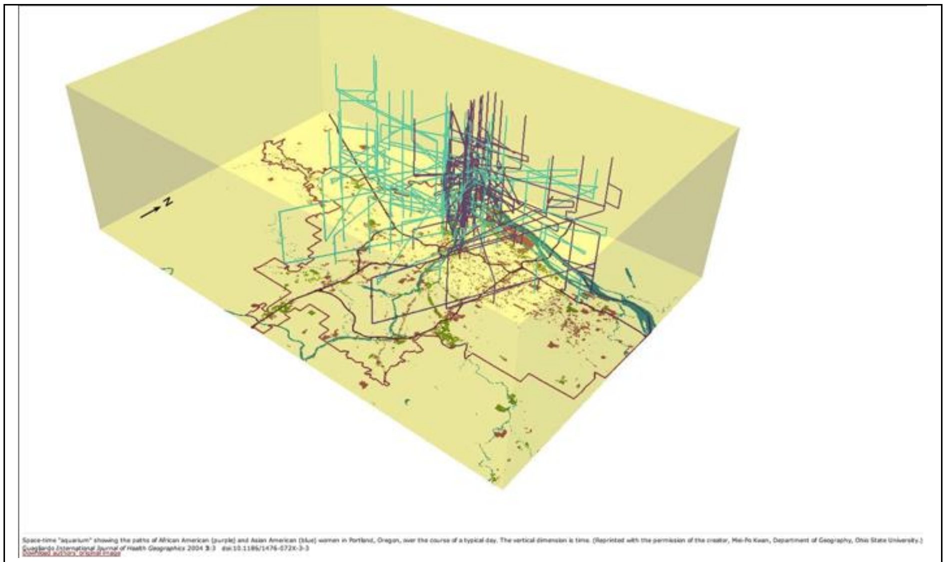


Figure 5. Space-time aquarium showing the space-time paths of African and Asian Americans in the sample (Kwan, 2004).

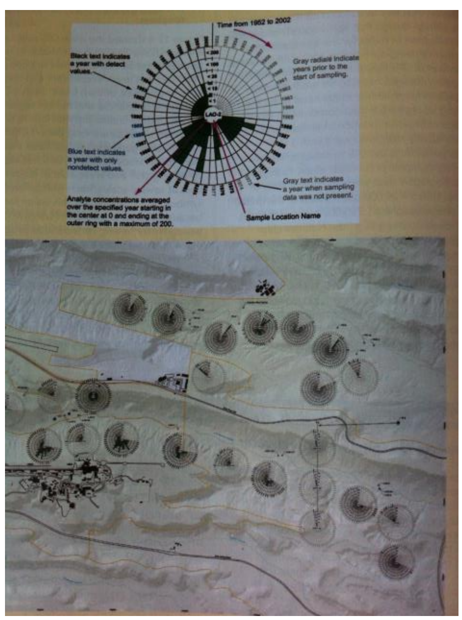


Figure 6. Map Clocks (Longley et.al. 2011, p.316)

Methodology

It all began with a very simple, hand drawn (**Figure** 7) , visual hypothesis.

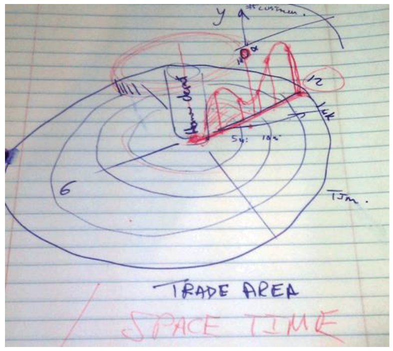


Figure 7. First Conceptualization

However "complexities" soon became apparent. There was a major incompatibility between the Huff (space) and Bass (time) models. Early attempts to resolve these differences are visualized in **Figure 8**.

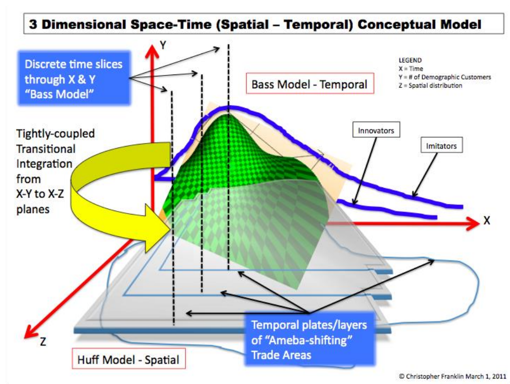


Figure 8. First Huff-Bass Conceptualization

Ultimately we formed an approach that included a transdisciplinary collaboration with computer science, natural science and mathematics, arts & humanities as well as many serendipitous inputs from various colleagues along the way and many conversational "*gedankenexperiments*"[1] before operationalizing the final Avatar.

Excel was used as a test bench to exercise ideas and concepts, after validating with brain science literature. For orientation in **Figure 9**, the x-axis from position (0, 0) represents the central cylinder or store in the final Avatar object.

[1] Thought experiments

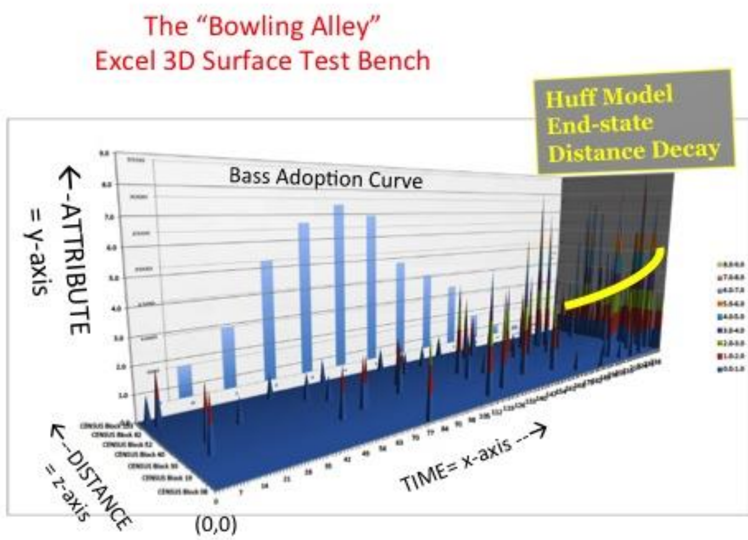


Figure 9. Early Excel 3D Surface Test bench for Prototyping the Space-Time Visualization

Building an Ensemble Space-Time Visualization Model

To this end we employ a Bayesian methodology to combine a spatial allocation approach inspired by the Huff gravity model with the Bass temporal product innovation model. The Huff model is spatial only, implicitly based on a future end-state or time "t_n".

In other words, it offers a snap shot of some future point in time where market penetration and market share reach equilibrium within a store level trade area (SLTA).

The Bass model on the other hand, forecasts the number of innovators and imitators over each time step in the product life cycle of new innovative products.

By combining the Huff inspired Bayesian methodology and the Bass model we create a basis for our space-time visualization model which potentially enhances understanding of the temporal and spatial interactions of sales event location, market penetration effects and trade area market share for new product introductions.

The steps through which we built the model follow.

Step One

The first step is to create a geospatial SLTA domain (**Figure 10**). We begin with a simple example of five (5) customers arrayed in a polar coordinate system with various bearings and Euclidean distances from the store located at the center.

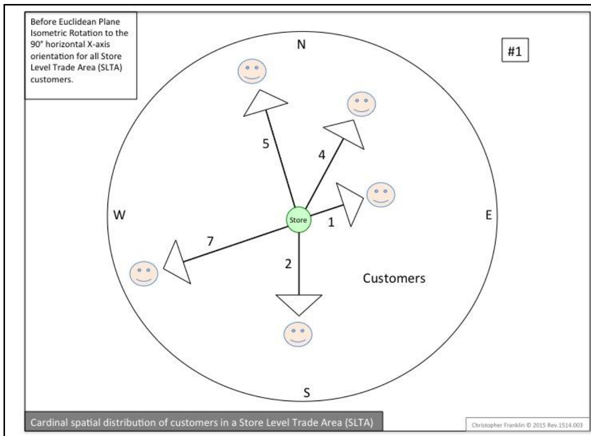


Figure 10. Begin with Geocoded Data

Step Two

Next we perform a rotational transformation of the vectors to one angle, 90 degrees in the horizontal (**Figure 11**). This new radius or summing vector has all of the unique customer Euclidean distances indicated on its line. Thus the radius becomes the x horizontal axis in the Avatar.

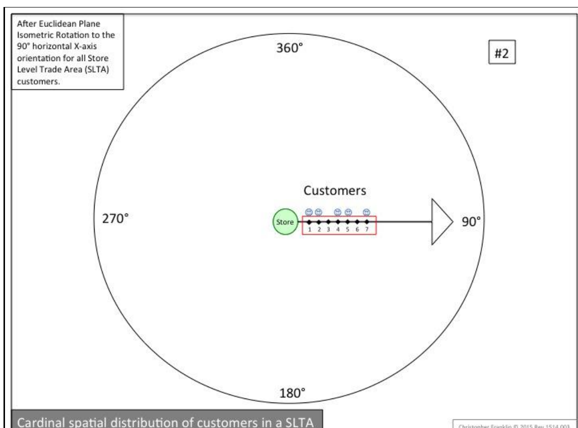


Figure 11. Rotate all distances to the X-axis

Step Three

Next we transpose from a compass rose to a clock face. **Figure 12** may assist the reader in visualizing this transformation and confirm the equivalent compass and time transpositions.

Figure 12. Compass Headings & 24 Hour Watch[2]

The final step is to add a multi-attribute vertical y-axis (**Figure 13**). We utilize a proportional measurement scale for the horizontal axis and vertical axis. The circumferential axis is in whatever time denominations deemed appropriate (e.g. years, months, days, hours and so on).

Finally, we are able to create two versions of our Avatar. The first is a physical 3-D printed object (**Figure 14**) and the second (**Figure 15**) is a virtual 3-D software object operating in EXCEL as a test bench. (The top transparent smaller disk is for a future use).

[2] Source: Public Domain https://commons.wikimedia.org/wiki/File:Direction_Watch.jpg

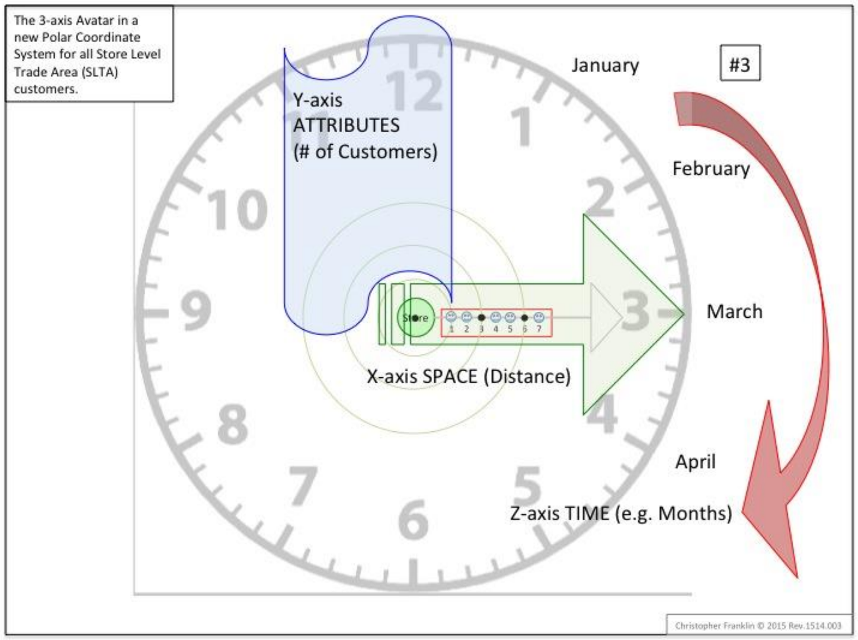


Figure 13. Core Space-Time Model

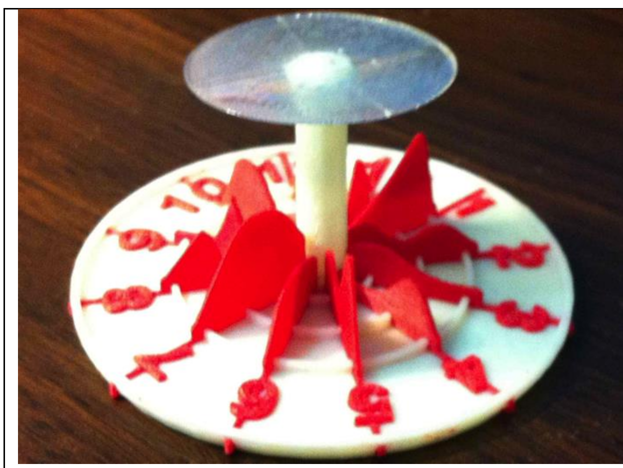


Figure 14. 3D Printed Avatar Model

Space distance (3,6,9 miles) + Time (12 mths) + Attribute (# of Customers)

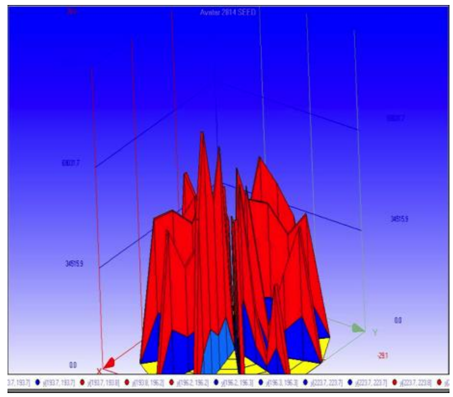


Figure 15. Spatial (x) + Temporal (z) + Attribute (y)

Findings

Informal presentations and focus groups with friends, colleagues and educated non-experts, at this stage of the research, phenomenalogically show the Avatar 3D printed model to be enthusiastically received as a visual-kinetic tool or object, with comments such as:

“After a brief explanation the Avatar is very easy to extract information from. It allows the visual learner to understand complicated data quickly.”[3]

“It materializes data to be analyzed or perceived with more than just the visual sense.” [4]

Focus group testing has been informal to date, but after only moments of viewing and handling the Avatar semaphore 3-D printed object device, respondents are eager to test their understanding by role playing as the store manager and developing scenarios that explain the likely reasons for the temporal and spatial variances and dispositions occurring as a result of hierarchical innovation diffusion.

This suggests that the approach of developing and generating, haptic visual and intuitively easy to understand, attribute data objects, across space-time continuums is plausible, feasible and of potential benefit to a wide audience of variously skilled practitioners and scholars.

With the advent of Very Large Data Repositories of unbounded and/or “countably infinite”[5] sized geo-big-data (referred throughout as ‘geo-big-data’); having the ability to compare scales of magnitude with ease as in “planets to atoms”, side-by-side, may provide additional important initial insights when beginning an investigation or study, as the burgeoning information age of the late 20^{th} century roars past midlife and beyond.

Additionally, a “normalizing constant” effect, across different scales of “big data” is easily visualized and inspected as to shape and structure (Mason, 1975). There are potentially many important implications for associated with developing new and meaningful visualization methodologies and models.

The normalizing constant feature of proportionality allows easy comparison of different scaled Avatars “side-by-side”, or through co-mingling of data in the same object, similar to the technique used in geoscience and referred to as “stratigraphy” (see the mocked-up visual of this below in Figure 16).

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[3] Christyna Marguerite

[4] Jose Manuel

[5] Definition - http://mathworld.wolfram.com/CountablyInfinite.html

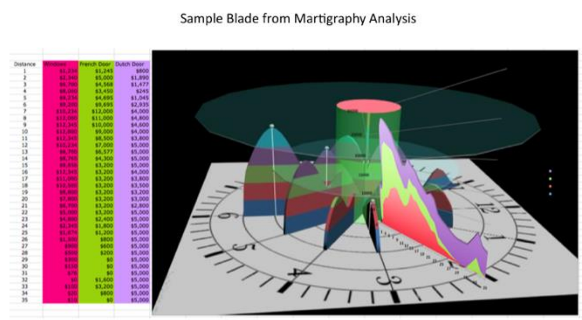

Figure 16. Mock-up example of "Stratigraphy" co-mingled in the Avatar

Summary

Moving location analytics past 2D/3D GIS symbolized maps, and toward space-time diffusion visualizations, using Bayesian inference, may hold potential for both increasing the quantity of "geo-big-data" that can be addressed while also increasing the degree of insight and understanding gained, through the simultaneous visuospatial harnessing of micro-analytic (Mason, 1975) and macro-analytic (Lusser's Law) retail trade area dynamics.

The complexity and volume of "geo big data" exposes one of geography's relative weaknesses, heavy reliance on map visualizations for all "geo-big-data". This may signal the need for ancillary and complimentary visualization methodologies, such as Avatar, to enable federated searches of "geo-big-data" events in space, time, as an exploratory technique, before engaging more detailed and traditional location analytic techniques.

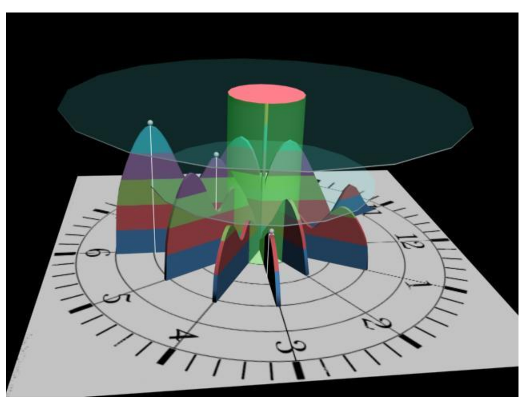

Figure 17. Future Avatar Semaphore Object (Time+Space+Attribute) modeled in Game Engine Software (UDK, 3DMAX, Unity etc.)

Finally, the 3-D printed version of the Avatar is also interesting for its "archeologic" or physical presence in terms of human perception (*i.e. Tactile and Haptic Perception*). By examining the Avatar physically, while viewing, discussing and hypothesizing, the brain collects a plethora of information about the object, much more than a 2D map, similar to the process an archaeologist goes through when finding a skull in the field. It is also common knowledge that Archaeologist even use tongue tests to determine the origin of certain objects.

Conclusion

The paper purposefully moves away from traditional geography visualizations of maps in a transdisciplinary fashion; and investigates the potential solutions for the display of "geo-big-data" events across space-time continuums while leveraging multidisciplinary knowledge of human perception. In so doing, we momentarily break the bonds of maps and compasses.

Adapting to visualizations of geo-big-data, as 3-D semaphoric objects will not initially be comfortable for some. However quantitative methods of the 1950's and 1960's faced the same challenges – initially.

By seeking the development of a novel visualization like Avatar, potential new insights may be achieved, thus strengthening the tools (both traditional and contemporary) available to support new directions of inquiry in support of improving location intelligence.

Human perception has a rich history in the psychological literature and the perception of objects is well documented. Transdisciplinary theories will and should play an important validating role in any innovative and potentially novel data visualization system. The Avatar, grounded in transdisciplinary visual-kinetic concepts, promises the potential of a standardized space-time modeling approach to meet new big data geovisualization needs.

The Avatar will identify spatial temporal consumer buying strength, leverage socio-demographic geo-segmentation targeting and facilitate fast and easy dynamic trade area analysis especially as an exploratory process to guide further GIS map based investigation.

Finally, the Avatar 3-D objects maintain a link to the GIS data structures (i.e. attribute table and geodatabase files) to enhance synergy.

Developing innovative "geo-big-data" visualizations could open knowledge pathways to ever-widening and expanding non-GIS audiences, from children to adults, from neophytes to experts (from many walks of life), all eager to easily consume and quickly understand growing volumes of attribute data of interest. The analogy comes to mind of those born in the mid to late 1800's, who eventually could speak with their grandchildren, after the invention of the telephone, without any knowledge of engineering, electricity or Morse code.

Limits and Delimits

The Avatar is limited to analysis of transactions where the customer geocoded residence addresses are available and Euclidean distance can be calculated from the residence to the store.

Avatar is purposefully limited to only the necessary and sufficient dimensional components sufficient and necessary for discovery of the spatial temporal skeletal framework of shape and structure (i.e. Time, Space and Attribute data)

Avatar visualizations depicted here are limited to our Alpha versions of 3D printed objects and software running in the Excel engine and game software engines UDK and 3DMAX, all of which are currently under further development.

The attribute data visualized is delimited to only those Census Blocks at each time step where the number of customers is greater then zero.

The Census Block population priors delimited to non-children (18+)

Visualizations are currently limited to inelastic demand.

Visualizations are limited to monopolistic competition

Space is delimited (ceteris paribus) to Euclidean distance only.

We specifically delimit distance to be defined as Euclidean (straight line) distance. There is an important rationale here for using Euclidean as opposed to shortest path or drive time. Unlike shortest path or drive time measurements, Euclidean distance measurements remain unchanged in growing urban landscapes and environments (e.g. changing road networks). Future work may be extended to include the addition of drive time and shortest path for possible additional perspective.

Future Visualization Development

The Avatar visualizations are being developed further, with game software quality imagery, environments and interactions, to strengthen the dynamic modeling depictions for Location Analytics and to improve high level Location Intelligence.

The game software quality of the Avatar visualization below in Figure 18 was rendered in 3D-MAX, Unity and UDK rendering engines.

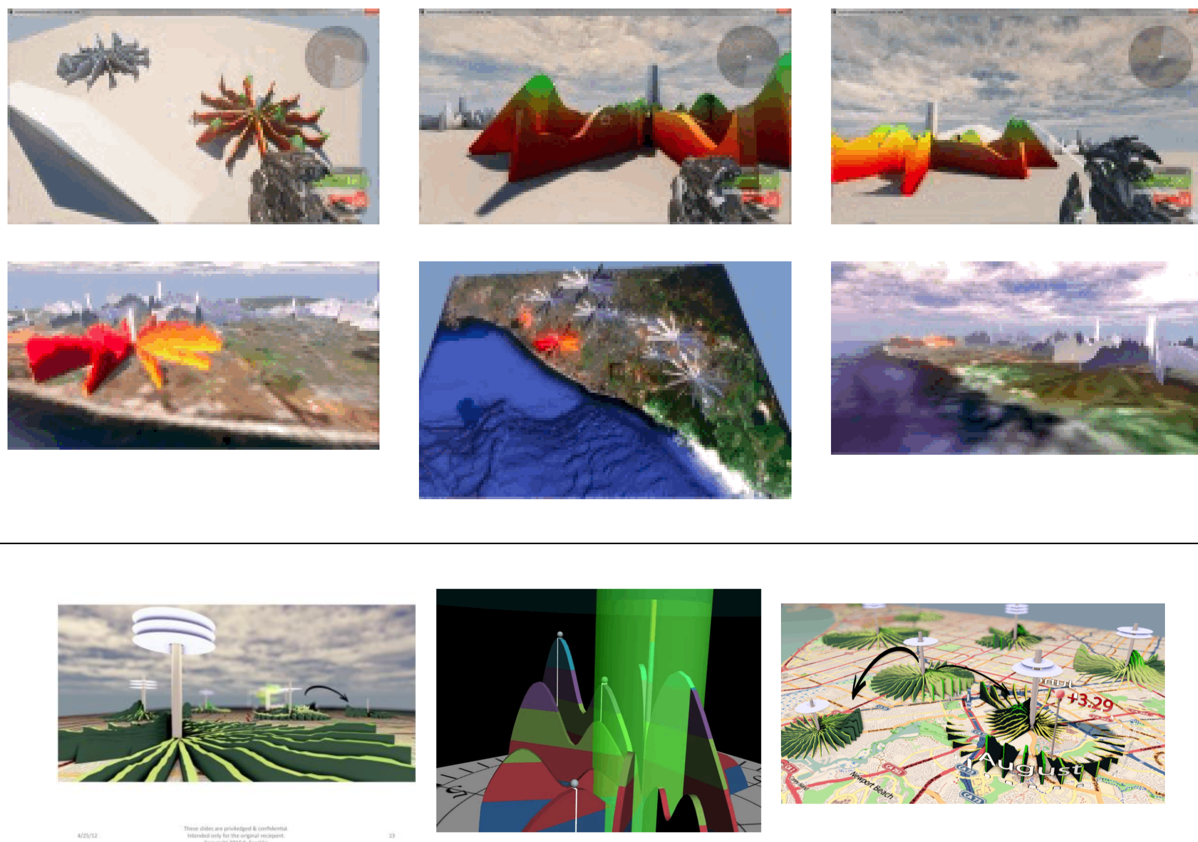

Figure 18. Game Software rendering of Stores in Orange County, CA

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